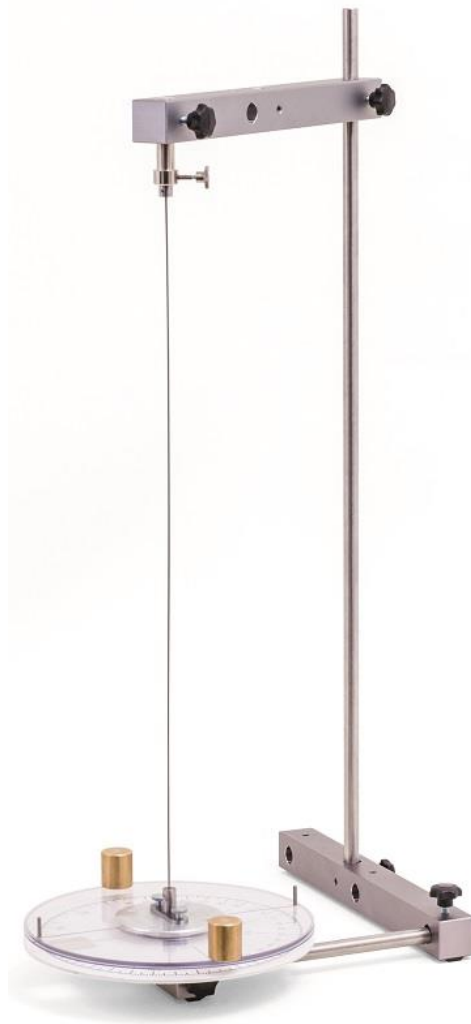


## Torsion Apparatus 1018550 Torsion Apparatus Supplement Set 1018787

### Instruction manual

11/15 TL/UD



### 1. Description

The torsion equipment is designed for determining the torsion coefficient and shear modulus of cylindrical metal wires by means of static measurements of the torsion angle and the force of torsion as well as dynamic measurements of periods of oscillation of a torsion pendulum.

The torsional force is conveyed via a pendulum disc to the clamped material sample. The angle and force resulting from the torsion can be measured with the help of a scaled disc and a spring dynamometer. The period of oscillation of the pendulum can be measured with the aid of a light barrier and a digital counter.

## 2. Equipment

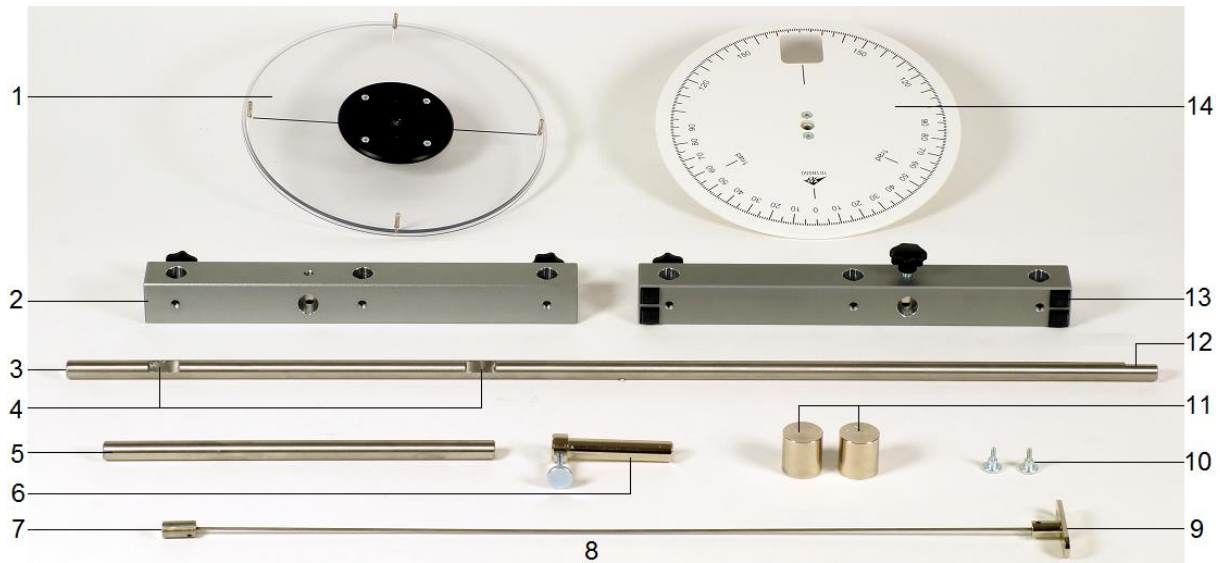


Fig. 1: Equipment included.

- |   |   |
|---|---|
| <ul style="list-style-type: none"> <li>1 Pendulum disc with 4 pins attached</li> <li>2 Upper cross piece</li> <li>3 Vertical rod</li> <li>4 Slot for upper cross piece (for cylindrical rods of length 500 mm or 300 mm)</li> <li>5 Horizontal rod</li> <li>6 Clamping sleeve</li> <li>7 Cylinder to fit clamping sleeve</li> </ul> | <ul style="list-style-type: none"> <li>8 Cylindrical steel wire (diam.: 2 mm, length: 500 mm)</li> <li>9 Clamping arm for cylindrical rod</li> <li>10 Fastening screws for clamping rod</li> <li>11 Weights</li> <li>12 Slot for lower cross piece</li> <li>13 Low crosspiece (with rubber feet)</li> <li>14 Scaled disc</li> </ul> <p>Base for light barrier 1000563 (not shown)</p> |
|---|---|

The apparatus features a scaled disc above which a transparent pendulum disc will rest. This is attached by means of a horizontal rod to a cross piece. A vertical rod is rigidly connected into the cross piece. A pendulum wire is suspended from another cross piece higher up the vertical rod by means of clamping sleeve. The vertical rod has slots cut into it to act as clamping surfaces for the cross pieces. This automatically ensures that the upper and lower cross pieces are attached in the correct alignment to one another. The pendulum disc has four pins

rising from it to accommodate pairs of additional weights placed symmetrically across the disc.

The material sample is a cylindrical steel wire with an arm at one end allowing it to be clamped to the pendulum disc and a cylinder at the other end to be clamped inside the clamping sleeve. The arm and the cylinder are both attached to the wire by means of two hex socket screws.

### 3. Additionally recommended

- |  |         |
|--|---------|
| Torsion apparatus supplement set   | 1018787 |
| Equipment included:  |         |
| 1 Cylindrical steel wire (diam.: 2 mm, length: 300 mm)                               |         |
| 6 Cylindrical wires made of brass/copper/aluminium (diam.: 2 mm, length: 300/500 mm) |         |
| 2 Cylindrical wires made of aluminium (diam.: 3/4 mm, length: 500 mm)                |         |

### 4. Other equipment required

- |                              |         |
|------------------------------|---------|
| 1 Precision dynamometer, 2 N | 1003105 |
| 1 Precision dynamometer, 5 N | 1003106 |
| 1 Light barrier              | 1000563 |
| 1 Digital counter @230 V     | 1001033 |
| or                           |         |
| 1 Digital counter @115 V     | 1001032 |

## 5. Technical data

### Torsion apparatus

Cylindrical wires

Material: Steel  
 Length: 500 mm  
 Diameter: 2 mm

Weights

Height: 27 mm  
 Diameter: 24 mm  
 Mass: 100 g  
 Dimensions: 700x400x400 mm approx.

Weight: 2.9 kg approx.

### Supplement set

Material	Diameter	Lengths
Brass	2 mm	300/500 mm
Copper		
Aluminium		
Aluminium	3/4 mm	500 mm



Fig. 3: Assembly including vertical rod.

## 6. Set-up

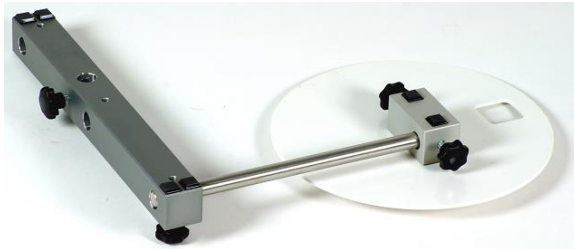


Fig. 2: Assembly of scaled disc with horizontal rod and cross piece.



Fig. 4: Assembly of upper cross piece (wire of length 500 mm), clamping sleeve and pendulum disc.



Fig. 5: Attachment of cylindrical wire to pendulum disc via clamping arm and with clamping sleeve placed over the cylinder.

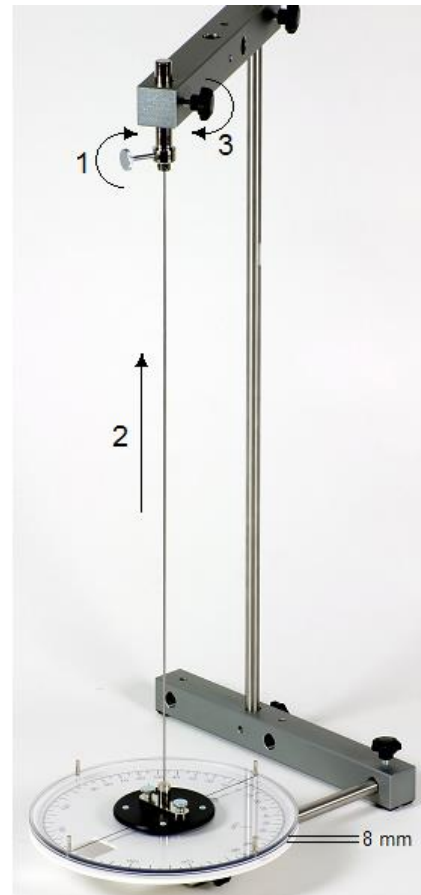


Fig. 6: Attachment of wire to clamping sleeve and adjustment of operating distance ( $\approx 8$  mm) between pendulum disc and scaled disc.

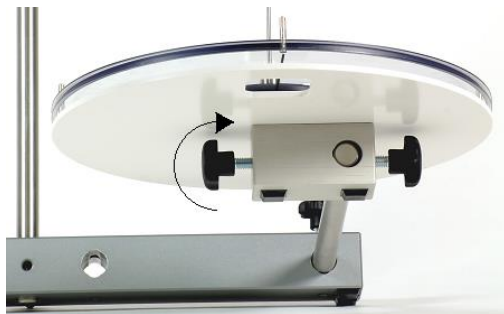


Fig. 7: Attachment of pendulum disc.

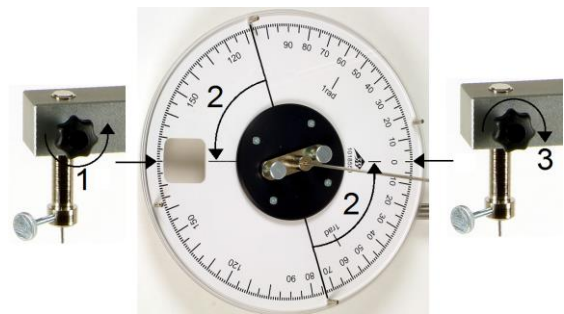


Fig. 8: Zero calibration of scaled disc.

### Interchanging cylindrical wires

Removing the cylindrical wires from the fully assembled apparatus is done by carrying out the steps shown in Fig. 6 and Fig. 5 in reverse. The replacement wires are inserted precisely as described in Fig. 5 and Fig. 6. Once the operating distance (step 2 in Fig. 6) has been calibrated once, it does not need to be recalibrated every time a new wire is inserted, since the pendulum disc cannot change position once it has been firmly fixed into place (Fig. 7).

## 7. Operation

### 7.1 Static measurement

For the static measurement, a force acting tangentially upon the pendulum disc is applied by means of a spring dynamometer.

- Calibrate the zero point for the 5 N spring dynamometer.
- Hook the 5 N spring dynamometer over the pin above the  $0^\circ$  mark on the scaled disc.
- Pull the dynamometer until the line marked on the pendulum disc coincides with the 1 radian mark on the scaled disc (Fig. 9). Make sure that the force that the dynamometer exerts on the pendulum disc acts at a tangent to the disc. The dynamometer must be aligned at  $90^\circ$  to the line marked on the pendulum disc.
- Read off the value of the force exerted from the dynamometer and write it down.

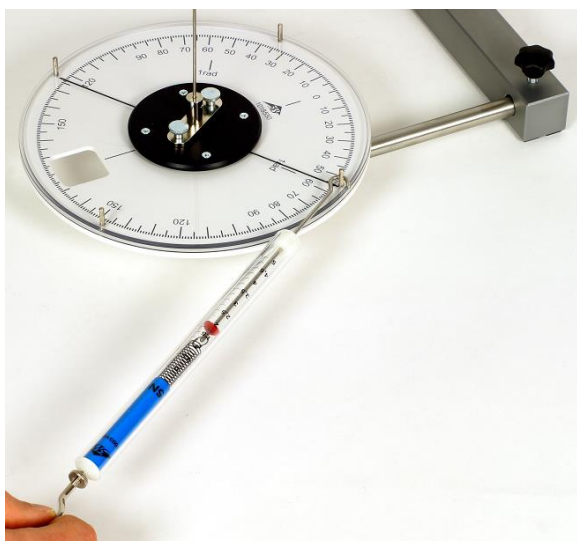


Fig. 9: Static measurement using a dynamometer.

#### Notes:

After each measurement, make sure that the line marked on the pendulum disc returns to coincide with  $0^\circ$  mark on the scaled disc. If not, the pendulum disc may need to be readjusted.

When using the cylindrical wires from the supplement set, it is recommended that a smaller deflection be used depending on the length and diameter of the wires.

### 7.2 Dynamic measurement

- Screw the light barrier 1000563 to its base. Position the light barrier and its base over the slot in the scaled disc slightly to the left or right of the line across the pendulum disc (Fig. 10).
- Connect the light barrier to input A of the digital counter. Turn the rotary selector knob for selecting the operating mode of the counter to the symbol for measuring the period of a pendulum.
- Deflect the pendulum disc without any weights added until the line marked across it coincides with the 1 radian mark on the scaled disc.
- Press "Start" on the digital counter and release the pendulum disc. Read off the first reading recorded for the period  $T_0$  from the digital counter's display and write it down.

#### Notes:

When using the cylindrical wires from the supplement set, it is recommended that a smaller deflection be used dependent on the length and diameter of the wires.

The torsional oscillations are heavily damped in a manner dependent on the length and diameter of the wire. It is therefore recommended that the first value recorded and displayed by the digital counter be used for the calculation of the results.

- Add the additional weights by placing them over the pins located over the  $90^\circ$ -markings on the scaled disc and repeat the measurement as described above. Read off from the digital counter the period  $T_{02m}$  for the torsion pendulum with two weights added and write it down.



Fig. 10: Dynamic measurement using light barrier and digital counter.

## 8. Sample measurement

Force $F$ required for dynamometer to rotate pendulum disc by 1 radian:	2.05 N
Period $T_0$ of torsion pendulum with no added weights:	461 ms
Period $T_{02m}$ of torsion pendulum with added weights:	767 ms

## 9. Evaluation

### 9.1 Moment of inertia of added weights

The added weights can be considered as solid cylinders to a quite good approximation since the holes for the pins on which they are placed are small enough to be ignored. The moment of inertia  $J$  for a solid cylinder is given by:

$$(1) J = \frac{1}{2} \cdot m \cdot r^2 .$$

$m$ : Mass of solid cylinder  
 $r$ : Radius of solid cylinder

Steiner's parallel axis theorem applies to the moments of inertia  $J_m$  of the added weights, since the weights oscillate around the axis of the pendulum at a distance  $R = 10$  cm:

$$(2) \begin{aligned} J_m &= J + m \cdot R^2 \\ &= \frac{1}{2} \cdot m \cdot r^2 + m \cdot R^2 = \frac{1}{2} \cdot m \cdot (r^2 + 2 \cdot R^2) . \end{aligned}$$

The moment of inertia  $J_{2m}$  of the two weights together is simply twice this:

$$(3) \begin{aligned} J_{2m} &= 2 \cdot J_m = m \cdot (r^2 + 2 \cdot R^2) \\ &= 100 \text{ g} \cdot ((12 \text{ mm})^2 + 2 \cdot (10 \text{ cm})^2) . \\ &= 0,002 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

### 9.2 Static measurement

At a distance  $R = 10$  cm from the axis of the pendulum, the dynamometer exerts a force  $F$  at a tangent to the edge of the disc, thus generating a torque  $M$ :

$$(4) M = R \cdot F .$$

The torque  $M$  is directly proportional to the deflection of the torsional pendulum by an angle  $\varphi$ . The constant of proportionality is the torsional coefficient  $D$ :

$$(5) M = D \cdot \varphi .$$

From equations (4) und (5) and the measurements made in section 8:

$$(6) D = \frac{R \cdot F}{\varphi} = \frac{10 \text{ cm} \cdot 2,05 \text{ N}}{1 \text{ rad}} = 0,205 \text{ Nm} .$$

The shear modulus  $G$  is a constant for a material which quantitatively describes the linear, elastic deformation of the material by a shearing force or tension. For a cylindrical wire of length  $L$  and diameter  $d$  it is as follows:

$$(7) G = \frac{2 \cdot L \cdot D}{\pi \cdot \left(\frac{d}{2}\right)^4} .$$

Therefore, for the steel wire:

$$(8) G = \frac{2 \cdot 500 \text{ mm} \cdot 0,205 \text{ Nm}}{\pi \cdot \left(\frac{2 \text{ mm}}{2}\right)^4} = 65,3 \text{ GPa} .$$

This result is of the same order of magnitude as the value quoted in literature ( $\approx 80$  GPa depending on the type of steel).

### 9.3 Dynamic measurement

In general, the period of oscillation  $T$  of a torsional pendulum is given by the following:

$$(9) \quad T = 2 \cdot \pi \cdot \sqrt{\frac{J}{D}} \Leftrightarrow D = 4 \cdot \pi^2 \cdot \frac{J}{T^2}.$$

$J$ : Moment of inertia

$D$ : Torsional coefficient

Since the moment of the pendulum disc is unknown, the torsional coefficient will be determined by measuring the periods of oscillation  $T_{02m}$  and  $T_0$  with and without added weights (s. 7.2 and 8), whereby the moment of inertia of the added weights is known (see 9.1). From equation (9):

$$(10) \quad D = 4 \cdot \pi^2 \cdot \frac{J_{2m}}{T_{02m}^2 - T_0^2}.$$

$J_{2m}$ : Moment of inertia of added weights

$T_{02m}$ : Period with added weights

$T_0$ : Period without added weights

Using the moment of inertia for the added weights calculated in section 9.1 and the measurements given in section 8 and substituting them into equation (10):

$$(11) \quad D = 4 \cdot \pi^2 \cdot \frac{0,002 \text{ kg} \cdot \text{m}^2}{(767 \text{ ms})^2 - (461 \text{ ms})^2} \\ = 0,210 \text{ Nm}$$

The shear modulus is derived from equation (7):

$$(12) \quad G = \frac{2 \cdot 500 \text{ mm} \cdot 0,210 \text{ Nm}}{\pi \cdot \left(\frac{2 \text{ mm}}{2}\right)^4} = 66,8 \text{ GPa}.$$

This result is of the same order of magnitude as the value quoted in literature ( $\approx 80 \text{ GPa}$  depending on the type of steel).

The values determined for the torsional coefficient  $D$  and shear modulus  $G$  by the static and dynamic methods are in agreement to within about 2%.

### 10. Storage, cleaning and disposal

- Keep the equipment in a clean, dry and dust-free place.
- Do not clean the unit with volatile solvents or abrasive cleaners.
- Use a soft, damp cloth to clean it.
- The packaging should be disposed of at local recycling points.
- Should you need to dispose of the equipment itself, never throw it away in normal domestic waste. Comply with the applicable regulations.



