Charging and Discharging a Capacitor

INVESTIGATION OF HOW THE VOLTAGE ACROSS A CAPACITOR CHANGES OVER TIME WHEN THE CAPACITOR IS CHARGING OR DIS-CHARGING.

- Measure the voltage across a capacitor as it charges and discharges when the DC supply voltage to a circuit is turned on and off.
- Determine the half-life period for charging and discharging.
- Investigate how the half-life period depends on the capacitance and the resistance.

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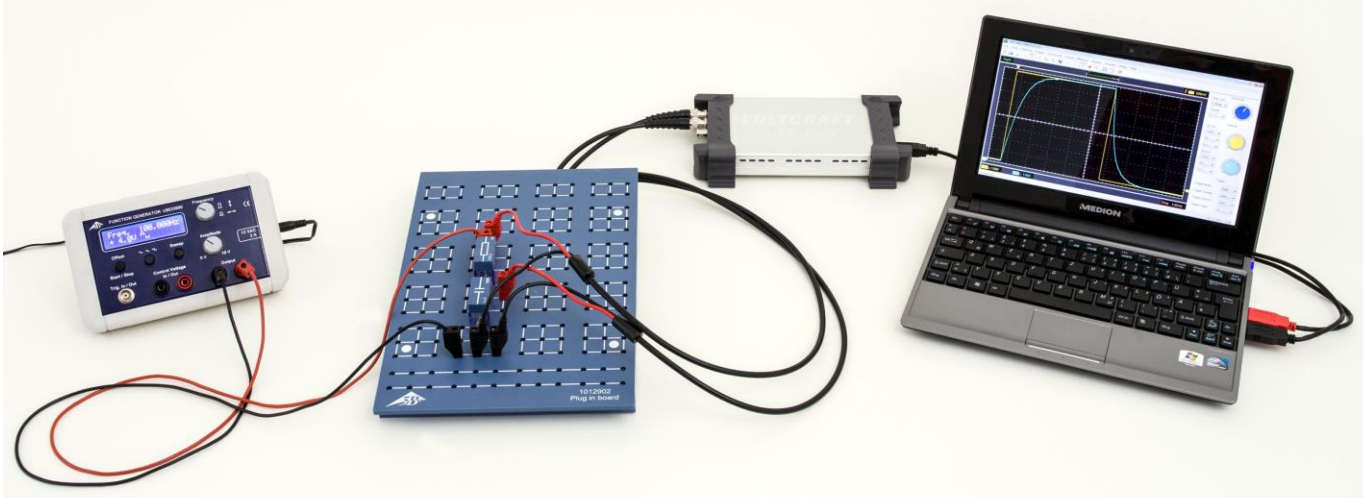

Fig. 1: Measurement set-up.

GENERAL PRINCIPLES

In a DC circuit, current only flows through a capacitor at the point in time when the power is turned on or off. The current causes the capacitor to charge up until the voltage across it is equal to the voltage applied. When the power is switched off, the capacitor will discharge till the voltage across it drops to zero. A plot of the capacitor voltage against time can be shown as an exponential curve.

For a DC circuit featuring a capacitance *C*, resistance *R* and a DC voltage U_0, the following applies when the supply is turned on:

$$(1)\quad U(t) = U_0 \cdot \left(1 - e^{-\frac{t \cdot \ln 2}{T_{1/2}}}\right)$$

The following applies when the power supply is switched off:

$$(2)\quad U(t) = U_0 \cdot e^{-\frac{t \cdot \ln 2}{T_{1/2}}}\,.$$

where

$$(3)\quad T_{1/2} = \ln 2 \cdot R \cdot C$$

$T_{1/2}$ is the half-life period, i.e. the voltage across a discharging capacitor will halve within a time $T_{1/2}$. The same period elapses when the voltage drops from a half to a quarter and from a quarter to an eighth.

These aspects will be investigated in the experiment. How the capacitor voltage changes over time is recorded using a storage oscilloscope. Since the DC voltage U_0 is set to 8 V, it is easy to read off a half, a quarter and an eighth of that value.

LIST OF EQUIPMENT

1	Plug-In Board for Components	U33250	1012902
1	Resistor 470 Ω, 2 W	U333022	1012914
1	Resistor 1 kΩ, 2 W	U333024	1012916
1	Resistor 2,2 Ω, 2 W	U333026	1012918
3	Capacitor 1 µF, 100 V	U333063	1012955
1	Function Generator FG 100	U8533600	1009956/7
1	USB Oscilloscope, 2x50 MHz	U112491	1017264
2	HF Patch Cord, BNC/4 mm Plug	U11257	1002748
1	Set of 15 Experiment Leads	U13800	1002841
1	Set of 10 Jumpers	U333093	1012985

Additionally required:

1 Computer running Win XP, Vista or Win7 operating system

SET-UP AND PROCEDURE

- Set up the apparatus for the measurement as shown in Fig. 2.
- Set up the circuit shown in Fig. 3 on the plug-in board.
- For the oscilloscope, select a time base of 1 ms, 1 V for the vertical deflection of CH1 and CH2, set the trigger mode to edge and and trigger sweep to auto. The source of the trigger should be CH1 and with a threshold of around 600 mV.

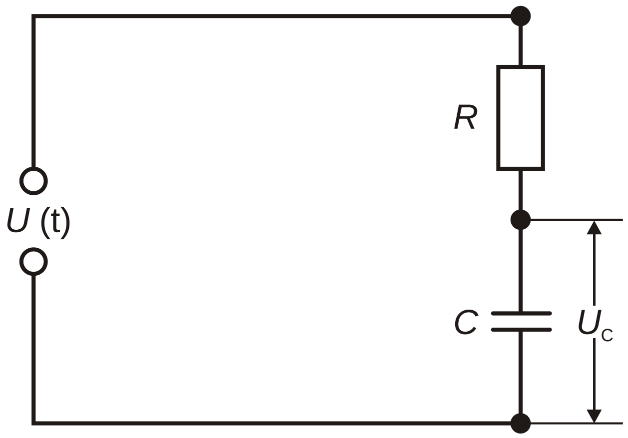

Fig. 3: Sketch of circuit.

Time to reach half-way when charging and discharging

- Plug a resistor of 1 kΩ and a capacitor of 1 µF into the plug-in board as sketched in Figs. 2 and 3.
- Set s frequency of 100 Hz on the function generator and selct a square waveform.
- Select an amplitude which will result in a signal with a peak to peak voltage $U = 8$ V, corresponding to ±4 divisions with a setting of 1 V/div.
- Read off from the oscilloscope the times $T_{1/2}$ it takes the voltage to halve as the capacitor discharges from voltages of $U = 8$ V down to $U = 4$ V, from $U = 4$ V down to $U = 2$ V and from $U = 2$ V down to $U = 1$ V and enter these values into Table 1.
- Read off from the oscilloscope the times $T_{1/2}$ it takes the voltage to reach the various half-way points when charging up to $U = 8$ V, i.e. $U = 0$ V up to $U = 4$ V, $U = 4$ V to $U = 6$ V and $U = 6$ V to $U = 7$ V. Enter these values into Table 2.

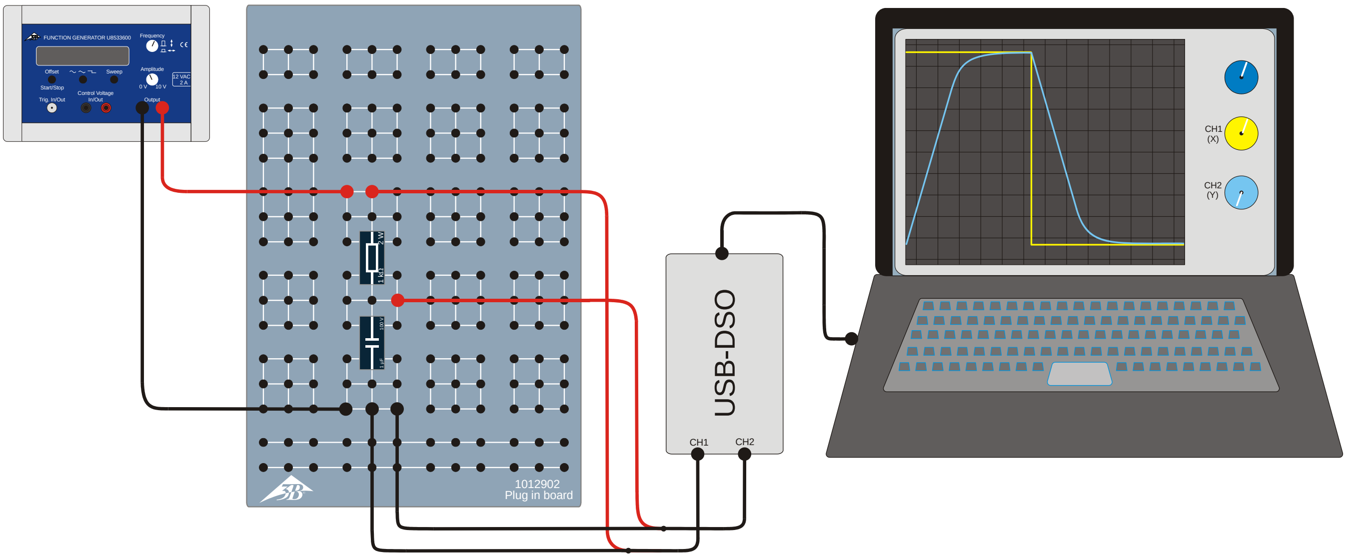

Fig. 2: Sketch of measurement set-up.

Half-way values for a fixed capacitor

- Use two capacitors with capcitance C = 1 µF in parallel to create a network with capacitance C = 0.5 µF.
- Set up circuits with resistance values of R = 0.47, 1, 1.47, 2.2 und 2.67 kΩ in succession. For those values which cannot be achieved using a single resistor, use two resistors in series.
- For each of these resistance values R, read off from the oscilloscope the time $T_{1/2}$ its takes the voltage value to reach half way and enter the results into Table 3.

Half-way values for a fixed resistor

- Use an ohmic resistor of value R = 470 Ω.
- Set up circuits with capacitance values of C = 0.33, 0,5, 0.67, 1, 1.5 und 2 µF in succession. For those values which cannot be achieved using a single capacitor, use multiple capacitors of value C = 1 µF in series or parallel.
- For each of these capacitance values, read off from the oscilloscope the time $T_{1/2}$ its takes the voltage value to reach half way and enter the results into Table 4.

SAMPLE MEASUREMENT

Fig. 4 shows an example oscilloscope trace for the charging and discharging of a capacitor. The yellow exhibits how the voltage across the ohmic resistance changes with time (CH1), while the blue curve represents the change in voltage across the capacitance (CH2).

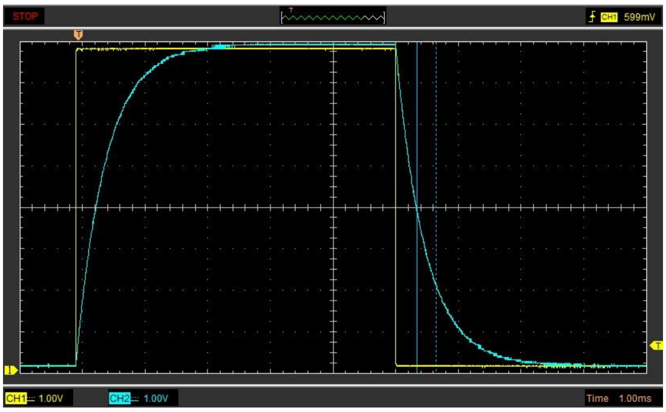

Fig. 4: Traces of voltage across a capacitor while charging and discharging recorded with an oscilloscope.

Tab. 1: Times $T_{1/2}$ for voltages to halve when a capacitor C = 1 µF discharges through a resistor R = 1 kΩ

$T_{1/2}(8\,\text{V} \to 4\,\text{V})$	$T_{1/2}(4\,\text{V} \to 2\,\text{V})$	$T_{1/2}(2\,\text{V} \to 1\,\text{V})$
0.69 ms	0.68 ms	0.70 ms

Tab. 2: Times $T_{1/2}$ for voltage to reach half way when a capacitor C = 1 µF charges via a resistor R = 1 kΩ

$T_{1/2}(0\,\text{V} \to 4\,\text{V})$	$T_{1/2}(4\,\text{V} \to 6\,\text{V})$	$T_{1/2}(2\,\text{V} \to 7\,\text{V})$
0.69 ms	0.69 ms	0.70 ms

Tab. 3: Times $T_{1/2}$ for voltage to reach half way when a capacitor C = 0.5 µF charges or discharges in circuits with various resistance values R.

$\frac{R}{\text{k}\Omega}$	0.47	1.00	1.47	2.20	2.67
$\frac{T_{1/2}}{\text{ms}}$	0.163	0.346	0.490	0.759	0.865

Tab. 4: Times $T_{1/2}$ for voltage to reach half way when capacitors C charge or discharge in circuits with a fixed resistance R = 470 Ω.

$\frac{C}{\mu\text{F}}$	0.33	0.50	0.67	1.00	1.50	2.00
$\frac{T_{1/2}}{\text{ms}}$	0.086	0.163	0.221	0.307	0.461	0.624

EVALUATION

Half-way values when charging and discharging

The agreement between the times it takes voltage to reach half way at various parts of the charging and discharging curves (Tables 1 and 2) confirms the expected exponential relationship, see equations (1) and (2).

Half-way value for a fixed capacitor

- Plot the “half-way” values $T_{1/2}$ from Table 3 against the resistance R on a graph and join up the points with a straight line through the origin (Fig. 5).
- To verify equation (3), determine the capacitance C from the gradient of the line m_C.

$$(4) \quad T_{1/2} = m_C \cdot R \text{ where } m_C = \ln 2 \cdot C$$

$$\Rightarrow C = \frac{m_C}{\ln 2} = \frac{0.33}{\ln 2} = 0.48\ \mu\text{F}\,.$$

The value determined by measurement is very well in agreement with the nominal capacitance value C = 0.5 µF.

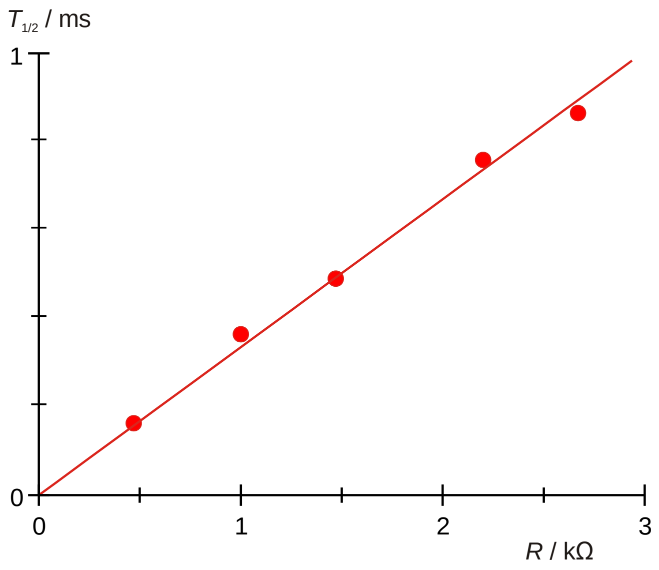

Fig. 5: Half-life $T_{1/2}$ as a function of resistance *R*.

Half-way value for a fixed resistor

- Plot the “half-way” values $T_{1/2}$ from Table 4 against the capacitance *C* on a graph and join up the points with a straight line through the origin (Fig. 6).
- To verify equation (3), determine the resistance *R* from the gradient of the line m_R.

$$(5)\quad T_{1/2} = m_R \cdot C \text{ where } m_R = \ln 2 \cdot R$$

$$\Rightarrow R = \frac{m_R}{\ln 2} = \frac{0.32}{\ln 2} = 0.46\ \text{k}\Omega\,.$$

The value determined by measurement is very well in agreement with the nominal resistance value *R* = 470 Ω.

- Multiply the values for the ohmic resistance *R* from Table. 3 with that of the capacitance *C* = 0.5 µF and alsoi multiply the capcitance values *C* from Table 4 with the resistance value *R* = 470 Ω. Enter these values for the product *R*·*C* alongside the corresponding “half-way” from Tables 3 and 4 into Table 5.

Tab. 5: Half-way times $T_{1/2}$ for products *R*·*C*, as calculated from the values in Rables 3 und 4.

$R \cdot C$ / kΩ · µF	$T_{1/2}$ / ms
0.16	0.09
0.24	0.16
0.24	0.16
0.31	0.22
0.47	0.31
0.50	0.35
0.71	0.46
0.74	0.49
0.94	0.62
1.10	0.76
1.34	0.87

- Plot the “half-way times $T_{1/2}$ from Table 5 against the prodcts *R*·*C* on a grph and join the points with a straight line through the origin (Fig. 5).
- To verify equation (3), determine the resistance use the gradient of the line *m* to determine that the coefficient of proportionality is ln2.

$$(6)\quad T_{1/2} = m \cdot R \cdot C \text{ where } m = \ln 2$$

The value obtained by measurement *m* = 0.67 is very well in agreement with the theoretical value ln2 = 0.69.

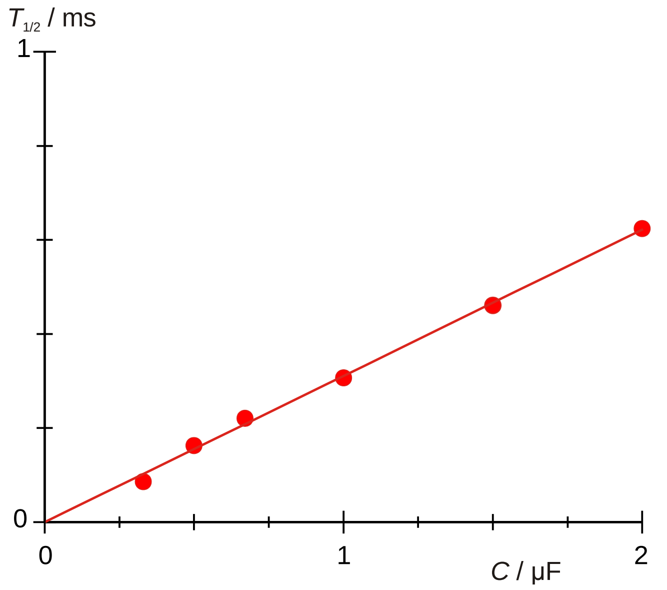

Fig. 6: Half-life $T_{1/2}$ as a function of capacitance *C*.

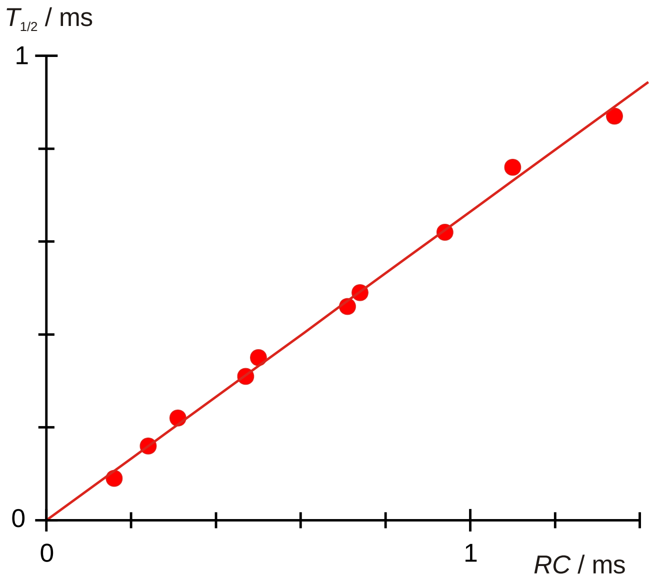

Fig. 7: Half-life $T_{1/2}$ as a function of the product of *R***C*.